Iterative Phase Retrieval

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Abstract

Over 25 years of phase retrieval is reviewed. Application areas include astronomy,[1,2] space-object imaging with both passive-incoherent[3] and active-coherent[4,5,6,7] illumination, wave-front and telescope-misalignment sensing,[8,9,10,11,12,13] 3-D coherent imaging,[14] and synthetic-aperture radar.[15,16,17] Algorithmic approaches include modifications of the Gerchberg-Saxton algorithm[18] such as the hybrid input-output algorithm,[1,19] gradient-search error-minimization techniques,[9,19] approaches to climbing out of stagnation,[20] support estimation from autocorrelation support,[21,22] phase diversity,[12,13] and sharpness maximization algorithms.[17]

Additional Comments

While projections-onto-sets algorithms can be used for phase retrieval, they perform poorly (except for easy cases) since the Fourier magnitude constraint is nonconvex. One can make the Fourier constraint convex by projecting onto the set of functions whose Fourier magnitude is less than or equal to the measured Fourier magnitude. However, that convex constraint does not sufficiently constrain the solution, giving rise to a uniqueness problem, as confirmed with computer simulation experiments during the conference. I suspect that just enforcing the zeros of the Fourier transform (another convex constraint) would fail for similar reasons. Other algorithms, such as hybrid-input-output and conjugate gradient search, are faster and more robust.

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